On the analysis of vertical circular cylindrical tanks under earthquake excitation at its base

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Abstract

Based on two international well accepted design standards, Eurocode 8 Part 4—Tanks, Silos and Pipelines and API Standard 650—Seismic Design of Storage Tanks, the structural response of seismically excited vertical circular cylindrical tanks is analysed from a novel perspective. The common basic assumption, adopted from Haroun-Housner and Veletsos, that a circular cylindrical tank containing liquid behaves like a cantilever beam without deformation of its cross-section is obsolete. Instead the authors consider the shell modal forms in order to generate a refined model. Emphasis is laid on the analysis of the fundamental frequencies for the tank-liquid-system. They are calculated by a new method, based on Galerkin’s approximations for cylindrical shells. As the results differ significantly from those calculated by the proposed formulae in both EC8 and API Standard 650, the new results are compared with tank failures during recent earthquakes. This comparison is astonishing. It can be seen from recent examples of tank damage that most failures are caused by resonance effects, which are taken into account neither in EC8 nor in API Standard 650. And, therefore, we take into account a high safety risk. This leads to the conclusion that the basic assumptions for current design provisions are no longer tenable under the present knowledge of shell theory and shell design, and, therefore should be reconsidered.

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1. Introduction

1.1. Objectives

Liquid storage tanks are very important components of industrial and agricultural facilities. The value of these tanks to society exceeds by far the economic value of the tanks and their contents. This is because the failure of tanks and their accessories is not limited to the immediate danger to nearby human lives, but also to a large extent leads to serious consequences and very likely to long-term environmental damages. The vulnerability of cylindrical tanks subjected to earthquake ground motion has been revealed in almost every major earthquake. In many of these earthquakes tanks have been severely damaged and some failed with disastrous consequences.

With the objective of improving the seismic safety and reducing the risk of damage or failure of thin walled cylindrical liquid storage tanks, numerous experimental studies and theoretical research projects have been carried out in recent years in order to better understand the behaviour of such tanks. Based on analytical results of Veletsos, Haroun and Housner, obtained from simplified beam models, and, in addition on experimental research, different simplified design provisions for anchored and unanchored tanks have been developed.

Such simplified design provisions can be found, for example, in the American Petroleum Industry (API) Standard 650 Appendix E—“Seismic design of storage tanks” [1] or in Eurocode 8—Part 4 “Tanks, Silos and Pipelines” in Appendix B “Seismic analysis procedure for anchored cylindrical tanks”[2].

Although these design provisions represent a significant advance in the design of tanks, observations in
recent earthquakes and experimental studies seem to indicate that they may not be adequate to accurately predict the seismic response of cylindrical tanks. Differences between the results of theoretical models, experimental studies and the real behaviour of storage tanks are often justified with the influence of material nonlinearities and/or structural imperfections and/or damping. Therefore, many research projects have been conducted in order to examine these influences. But up to now, to the knowledge of the authors, no satisfactory results have been obtained. This fact motivated the authors to analyse seismically excited liquid storage tanks from a new point of view.

1.2. Procedure

To give an insight into current design standards, a short comment about API Standard 650 and Eurocode 8 is given in Section 3. Two basic assumptions of the current design provisions, which are based mainly on the research work of Haroun–Housner, Veletsos and others are:

- The tank oscillates relative to the base as “vertical cantilever beam”.
- The initial circular cross-section maintains circular (no ovalization).

These assumptions are not adopted. Instead in the following sections a new method with essential novel ideas is presented.

In Section 4 a simple and convincing analysis is presented in order to determine the fundamental frequencies of empty tanks. The fundamental natural frequency $f_0$ of the tank is defined as the lowest value of frequency associated with a natural mode of vibration. Comprehensive nomograms are presented which provide the natural frequencies for circular cylindrical shells of any height, radius and wall thickness for any corresponding transverse modal shapes.

In Section 5 the obtained results are transferred to a new model in order to determine the natural frequencies of the tank–liquid-system (like a deformable shell), avoiding the use of the “cantilever beam model”. To check the efficiency and applicability of the new model the new frequencies are compared to those proposed in both, Eurocode 8 and API Standard 650, in order to draw consequences for these design provisions. Finally they are analysed in Section 7 with regard to tank failures of recent earthquakes.

Further a deformability parameter $K$ [Eq. (6)] is introduced, and it is shown that this parameter is an excellent indicator for the resistance of tanks, subjected to seismic actions.

As the proposed new method and its theoretical background are different to all existing methods and assumptions of tank design, the methodology and the applicability of the presented new method will be discussed intensively in Section 9. Justification of the proposed method relies on experimental work of Clough [12] and on former theoretical work by Urrutia Galicia in the fields of stresses, stability and frequency analysis of thin circular cylindrical shells [7,13,14].

2. Mathematical fundamentals

The basic system investigated is shown in Fig. 1. It is a ground supported, anchored, upright, circular cylindrical tank of radius $r$, height $L$, and wall thickness $h$. It is filled with liquid to the height $H$. The tank is excited horizontally by the seismic ground acceleration $x(t)$, its associated velocity $\dot{x}(t)$ and its corresponding displacement $x(t)$. The locations of points of the tank are defined by the cylindrical co-ordinate system $(r, \Theta, z)$, with the origin at the centre of the tank base, and $\Theta=0^\circ$ in direction of the horizontal excitation, as shown in Fig. 1. It is assumed that the liquid is incompressible and inviscid, that the flow is irrotational, and that all structural and liquid motions remain within the linear elastic range of response.

3. Eurocode 8 and API Standard 650

Both, Eurocode 8—Part 4 and API Standard 650, represent an international well accepted design philosophy. The API Standard 650 Appendix E—“Seismic design of storage tanks” is the current design standard in the United States of America, but it is also used in Mexico and in most of the countries of Central and South America.
America. The ENV 1998-4 (Eurocode 8) is still a
pre-standard for the not yet released new European standard.
Therefore, it has not been the topic of many discussions
nor has it proved its worth in current practice. As with
most design provisions in structural engineering, both
standards comprise a lot of research results, which have
been gained over many years. These results are evalu-
ated and simplified to few empirical equations, what
leads to the following problem:

Although it is a very simple matter to deal with the
design provisions, it is a difficult or even an impossible
task to understand the applied formulae and to gain
further knowledge about their background. This state-
ment is especially true for the API Standard. Eurocode 8
gives more insight into its background, but leaves many
uncertainties to the user in questions of applicability.

Both, EC8 and API Standard use nearly the same
approaches to include the time dependent behaviour of
the liquid. They can be explained as follows: One portion
of the liquid along the walls and the bottom moves
in unison with the tank as a rigidly attached mass. The
other portion moves independently, experiencing slosh-
ing or rocking oscillations. This latter portion undergo-
ing sloshing motion is known as the “convective” or
“sloshing” mass component, whereas the other portion
of liquid moving synchronously with the tank is termed
as the “impulsive” mass component. Both components
cause a pressure loading. These two pressure compo-
ents are considered in both design provisions and are
expressed by the same equations.

In Eurocode 8 a third pressure component is taken
into account. This contribution is called “the im-

tulsive effect due to tank deformation”. According to the
meth-
odology of Eurocode 8 “fluid particles oscillate rigidly
with the tank in its deformation relative to the base as
a vertical cantilever”.

This third pressure component is not specifically
regarded in the API Standard as an additional mass,
because according to Veletsos and Yang and Haroun and
Housner the pressure distributions for rigid and flex-
ible tanks are very much alike, as indicated in Fig. 2 [11].
The pressure distributions for the flexible tanks (in Fig.
2 for h/R≤0.01) only differ significantly from rigid tanks
in the lower part of the tank wall.

But even if the pressure distribution of the hydro-
dynamic wall pressure is considered to be independent of
the wall flexibility, the authors note that the magnitude
of the pressure is highly dependent on the wall flexi-

bility. This is proved by the fact that the resulting pres-

sure is calculated by multiplying the pressure distri-

bution function by the spectral value of the pseudo-acce-

leration function A(t), instead of the ground acceleration \( \bar{a}(t) \), as
it is done for rigid tanks. The acceleration response func-

tion represents the instantaneous value of the pseudoac-

celeration induced by the seismic movements in a single
degree of freedom oscillator, having the natural fre-

quency and damping of the first sloshing mode.

If the fundamental natural frequency of the tank-liquid
system falls in the amplified acceleration region of the
design response spectrum, the spectral value of A(t) will
be significantly greater than the ground acceleration.
This leads to a higher corresponding maximum wall
pressure than that given by the rigid tank solution.

Therefore in both design provisions the knowledge of
the fundamental frequency \( f_0 \) of the tank liquid system
is necessary in order to

(a) calculate the wall pressures, and
(b) avoid resonance effects.

In Eurocode 8 the following approximation is given
for the fundamental frequency \( f_0 \), valid for the case of a
rigid soil:

\[
f_0 = \left[ \frac{E_s h^2 \frac{z}{h} = \frac{1}{3}}{2 \cdot r \cdot g(\gamma)} \right]^{1/2} - 0.15 \cdot \gamma + 0.46 \text{ and } \gamma = \frac{H}{r}
\]

where \( E_s \) is the Young’s modulus of elasticity for the
material of the tank wall, \( \rho_L \), the mass density of the
liquid, \( H \) the height of the liquid, \( h \) the tank wall’s thick-
ness at one-third of the tanks height, and \( r \) is the tank’s
radius. A governing parameter is \( \gamma \), which is the ratio of
liquid height \( H \) to the radius of the tank \( r \).

In the API Standard the fundamental frequency \( f_0 \) is
empirically calculated by

\[
f_0 = \frac{C_1 \cdot 1}{2 \pi H \sqrt{\rho_s}}
\]

where \( E_s \) is the Young’s modulus of elasticity for the
material of the tank shell, \( \rho_s \), the mass density of the tank
wall, and \( H \) the height of the liquid. The dimensionless
coefficient \( C_1 \) depends on the tank proportions, \( L/r \) and
\( h/r \), on Poisson’s ratio for the tank material \( v \) and on the
relative mass densities of liquid and tank walls.

Values for the coefficient of \( C_1 \) are presented for
example by Veletsos and Shivakumar [3]. Complemen-
tary data were obtained by Haroun and Housner [4].

As the presented expressions for the fundamental fre-

quencies were found independently in different research
projects applying different methods, these results cer-

tainly appear convincing. However, all these results are
based on the same assumption that the tank vibrates in
a combination of the following modes (Veletsos and
Yang, 1977) [16]:

\[
E_s \cdot h^2 \left( \frac{z}{h} = \frac{1}{3} \right) \quad \frac{1}{2 \cdot r \cdot g(\gamma)} - 0.15 \cdot \gamma + 0.46 \quad \text{and} \quad \gamma = \frac{H}{r}
\]
1. As a cantilever flexural beam, without distortion of its cross-section,
2. as a cantilever shear beam, again without distortion of its cross-section, and

Bearing in mind that all the presented results are based on the same assumptions and have the same background, it is not surprising that the achieved results for the fundamental frequencies equal each other, although computed by different formulae.

The assumed “vertical cantilever beam” model might be acceptable for a very tall, stiff and slender tank. However, regarding a tank of typical practical proportions with for example a diameter of 54 m, a height of 14 m, and a wall thickness of 31 mm, it is hard to imagine this tank behaving like a cantilever. The shell modal forms should be taken into account for such flexible tanks. The theory we are dealing with is based on a model towards more realistic results. However, analysing the fundamental modal forms of a shell instead of using the cantilever beam model, we are (as it will be shown further on) actually dealing with a completely different set of natural modes and frequencies for the coupled tank fluid systems.

4. Empty tanks

As a first step the frequency analysis can be performed for the empty tank, analysing the modal forms of a shell. Flexible normally loaded circular cylinders tend to vibrate in two directions, expressed by the axial wave number $m$, and the circumferential wave number $n$, depending on the frequency of excitation.

The first axial and circumferential modal forms are shown in Fig. 3. According to the Donnell–Mushtari–Vlasov Equations, solved by Galerkin’s approximate method [5], the following approximation can be used to determine the natural frequencies $\omega_{mn}$ for shells with different boundary conditions for every combination of axial wave number $m$ and circumferential wave number $n$:

$$\omega_{mn}^2 = \frac{1}{\rho_s h} \left[ \frac{E_s h \lambda_m^4}{r^2 \left( \frac{n}{r} \right)^2 + \lambda_m^2} \right]^2 + D \left[ \frac{n^2}{r^2} + \lambda_m^2 \right]^2$$

(3)

where: $E_s$=Young’s modulus for the material of the shell, $D$=flexural rigidity (bending stiffness) for the homogeneous isotropic case, expressed as

$$D = \frac{E_s h^3}{12(1-\nu^2)}.$$  

(4)

$\rho_s$=density of the shell material (steel), $\lambda_m$=Eigenvalues for longitudinal wave numbers of beams with different boundary conditions. For the simply supported case they can be expressed as

$$\lambda_m = \frac{m \pi}{L}.$$ 

(5)

The parameters $h$, $r$, $n$ have been explained before (Figs. 1 and 3)

For other boundary conditions than the simply supported case, $\lambda_m$ can also be approximated by the characteristic values of the corresponding beam problem, divided by $L$, listed for example in [6], and in Table 1.

The following assumptions are made to obtain the Donnell–Mushtari–Vlasov equations:

1. The contributions of in-plane deflections can be neg-
Axial wave numbers:

\[ m = 1 \quad m = 2 \quad m = 3 \quad m = 4 \]

Circumferential wave numbers:

\[ n = 0 \quad n = 1 \quad n = 2 \quad n = 3 \quad n = 4 \quad \text{etc.} \]

Fig. 3. Axial and circumferential modal forms.

Table 1

<table>
<thead>
<tr>
<th>Boundary conditions (base/top)</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
<th>( m = 4 )</th>
<th>( m = 5 )</th>
<th>( m &gt; 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free/free</td>
<td>4.730040</td>
<td>7.853204</td>
<td>10.99560</td>
<td>14.13716</td>
<td>17.27875</td>
<td>((2m+1)\pi/2)</td>
</tr>
<tr>
<td>Clamped/free</td>
<td>1.875104</td>
<td>4.694091</td>
<td>7.854757</td>
<td>10.99554</td>
<td>14.13716</td>
<td>((2m-1)\pi/2)</td>
</tr>
<tr>
<td>Free/simply supported</td>
<td>3.926602</td>
<td>7.068582</td>
<td>10.21017</td>
<td>13.35176</td>
<td>16.49336</td>
<td>((4m+1)\pi/4)</td>
</tr>
<tr>
<td>Both edges simply supported</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( m\pi )</td>
</tr>
<tr>
<td>Clamped/simply supported</td>
<td>3.926602</td>
<td>7.068582</td>
<td>10.21017</td>
<td>13.35176</td>
<td>16.49336</td>
<td>((4m+1)\pi/4)</td>
</tr>
<tr>
<td>Clamped/clamped</td>
<td>4.730040</td>
<td>7.853204</td>
<td>10.99560</td>
<td>14.13716</td>
<td>17.27875</td>
<td>((2m+1)\pi/2)</td>
</tr>
</tbody>
</table>

lected in the bending strain expressions but not in the membrane strain expressions.

2. The influence of inertia in the in-plane direction is neglected; therefore the theory is restricted to normal loading.

3. The shear terms are neglected.

According to these theories numerical solutions have been derived by Urrutia-Galicia explicitly explained in [7] where different nomograms have been presented to obtain the fundamental frequencies of circular cylindrical shells. Further it is shown that with regard to the frequency equation for circular cylindrical shells, there exists a quantity \( \Omega = (\omega_{mn} r)^2 \) which must remain constant and invariant once we fix the slenderness and the thickness ratios \( L/r \) and \( r/h \). The quadruple logarithmic coupling among the parameters \( \omega, L/r, r/h \) and the axial wave number \( m \) is fully displayed by means of a set of parallel straight lines, as shown in Fig. 4.

As seen in Fig. 4, circular cylindrical shells were analysed for height to radius ratios \( L/r \) from 2 to 100, and a radius to wall thickness ratio \( r/h \) from 20 to 500. Most of the established liquid tanks have a \( L/r \) ratio in a range from 0.3 to 3, and \( r/h \) ratios in a range from 250 to 1000.
Therefore, in reference [8] equivalent analyses have been carried out for tanks with small $L/r$ and high $rh$ ratios and it is shown in corresponding nomograms, that also for these lower ranges convergence is given.

Most of the existing literature dealing with cylindrical structures characterises them using the well-known ratios height to radius $L/r$ and thickness to radius $h/r$. There is, however, an alternative approach, which combines both relationships in a single deformability parameter $K$, defined as

$$K = \frac{L}{r} \sqrt{\frac{h}{r}} \quad (6)$$

This is a shell indicator that characterises certain classes of cylinders which behave the same while vibrating or buckling. In Section 7 it can be shown that the parameter $K$ is a good indicator for the earthquake resistance of liquid filled tanks as well.

It is further shown in [7,8] that not only the critical natural frequency of a cylinder depends on the deformability parameter $K$, that also the number of waves “$n$” of the critical mode depends on the deformability parameter $K$.

Experimental evidence [9] in the field of stability has proved the same dependence between the $K$ parameter and the slenderness ratio $L/r$, with stability predictions of up to 95% accuracy.

To illustrate the connection between natural frequencies and their corresponding modal forms, an existing petroleum tank with a capacity of 32,000 litres is calculated with the following parameters:

$$E_s = 2.06E + 05 \text{ N/mm}^2, r = 27 \text{ m}, L = 14 \text{ m},$$

$$h = 31 \text{ mm}, \mu = 0.3, \rho_s = 7.85E – 09 \text{ Ns}^2/\text{mm}^4.$$

Applying Galerkin’s Eq. (3) the natural frequencies are achieved for different axial modes $m$, and different circumferential modes $n$ (Fig. 5). It is further shown that there is a fundamental circumferential wave number “$n$” for a minimum frequency corresponding to every axial mode $m$. For example, for $m=1$ wave number $n$ is 13. The corresponding fundamental modes and their frequencies are listed in Table 2.

### 5. Liquid containing tanks

As none of the existing steel liquid tanks can be regarded as rigid, and few of them exceed $L/r$ ratios of 3, the presented cantilever beam model is abandoned, and the emphasis is laid on the frequency analysis as presented for empty tanks, also valid for buckling design.

The difficulty is how to include the liquid’s mass in the analysis of the fundamental frequency. One possible model would be to analyse the modal shapes considering the inertia forces generated by the contained liquid as an external force acting on the tank wall. Obviously this would be a very complicated solution, because the boundary conditions for the contained liquid would change with every different modal shape, and, therefore, it would also be very difficult to apply the fluid pressures correctly to the shell.

The idea of the new model is to attach the impulsive liquid mass component uniformly to the tank wall, resulting in a new effective density of the wall. With the new effective density the fundamental frequencies can be calculated by the same formula as presented for empty tanks.

The new effective density of the tank wall can be calculated by the following equation:

$$\rho^*_s = \frac{\text{liquid mass} + \text{tank wall mass}}{\text{tank wall volume}}$$

$$= \frac{(\pi r^2 H \rho_s) + (2 \pi r h L \rho_s)}{2 \pi r h L} \quad (7)$$

All the computed results presented in the following tables and figures are valid for roofless anchored tanks filled with water, with a shell density $\rho_s=7850 \text{ kg/m}^3$, and a liquid density $\rho_L=0.127*\rho_s$. All the results are obtained for a completely filled cylindrical tank clamped at one edge. For our example we choose $H=L=10 \text{ m}$ (fully filled).

In Table 3 the fundamental frequencies of the tank–

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### Table 2

<table>
<thead>
<tr>
<th>Axial wave number $m$</th>
<th>Circumferential wave number $n$</th>
<th>$f$ in Hz (minimum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>2.89</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>7.21</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>12.06</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>16.88</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
<td>21.70</td>
</tr>
</tbody>
</table>

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Fig. 5. Natural frequencies for any axial and circumferential wave numbers “$m$” and “$n$".
Table 3
Fundamental frequencies—Veletsos and new model

<table>
<thead>
<tr>
<th>Tank parameters:</th>
<th>Veletsos: (filled tank)</th>
<th>New model:</th>
<th>Deviation:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fund. mode Empty Filled</td>
<td>f(1) (Min)</td>
</tr>
<tr>
<td>L/r [-]</td>
<td>h/r [-]</td>
<td>K [-]</td>
<td>f0 [Hz]</td>
</tr>
<tr>
<td>1</td>
<td>0.0005</td>
<td>0.150</td>
<td>5.055</td>
</tr>
<tr>
<td>0.7</td>
<td>0.001</td>
<td>0.149</td>
<td>6.515</td>
</tr>
<tr>
<td>0.5</td>
<td>0.002</td>
<td>0.150</td>
<td>8.308</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.211</td>
<td>5.194</td>
</tr>
<tr>
<td>1.4</td>
<td>0.001</td>
<td>0.210</td>
<td>7.468</td>
</tr>
<tr>
<td>1</td>
<td>0.002</td>
<td>0.211</td>
<td>10.037</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.259</td>
<td>4.590</td>
</tr>
<tr>
<td>2.1</td>
<td>0.001</td>
<td>0.258</td>
<td>7.232</td>
</tr>
<tr>
<td>1.5</td>
<td>0.002</td>
<td>0.259</td>
<td>10.485</td>
</tr>
</tbody>
</table>

Comparing the frequencies proposed by Veletsos with the frequencies calculated by the new model, we gain the following results:

1. The frequencies $f_i(n=1)$ for the first circumferential modal shapes (filled tank), calculated by the new model are all about the same value (1.5 times) higher than those proposed by Veletsos ($f_0$). It shows, that the new model considers very precisely the influences of different tank/liquid heights and its corresponding impulsive mass components.

2. As expected from the empty tank analyses, also at the filled tanks there are much smaller frequencies relating to higher circumferential modal shapes.

These conclusions can be illustrated with a simple example: a liquid containing steel tank, completely filled with water, with the following parameters is given:

Liquid Height: $H = L = 10$ m,
shell density: $\rho_s = 7.85 \times 10^3$ kg/m$^3$,
tank radius: $r = 20$ m,
wall thickness: $h = 2$ cm.
frequency \( f_1(\text{Min}) \) for different tank dimensions. As a conclusion of the presented results, the aim of future research projects must be to avoid the use of the beam model, and to focus on the fundamental frequencies, which obviously are not related to the first fundamental modes of a cantilever beam, but rather to the fundamental modes in the range of \( 5 < n < 25 \), depending on the deformability parameter \( K \).

6. Experimental achievements

Although the existence of the so-called \( \cos n\Theta \)-type modes \((n=1 \text{ to } \ldots) \) (Fig. 7) has been proven in the literature, most of the current authors do adopt the beam model and even recommend that only the influence on the \( \cos \Theta \)-type mode \((n=1) \) should be investigated, with the reason that earthquake motions tend to strongly excite only this mode.

The latter assumption is questionable, because most of the existing steel tanks are very flexible, and the experimental analysis of steel tank models [12] showed that earthquakes strongly excite the shell buckling modes (\( \cos n\Theta \)-type) instead of the \( \cos \Theta \)-type beam mode.

In reference [12] different specimens of tanks are presented that were tested on a shaking table at the University of California, Berkeley, Richmond Field Station. The shaking table was a 6 m \( \times \) 6 m concrete slab, heavily reinforced and post-tensioned in both directions weighing about 45 t. The table was driven by hydraulic actuators able to move the table simultaneously in vertical and horizontal direction, according to any digitised accelerogram. One of the analysed specimens is a 1/3-scale model of a 11 m diameter, by 5.5 m high, steel prototype. The model was made out of two aluminium rings, and had the following parameters:

\[
L = 1.82 \text{ m},\ r = 1.82 \text{ m},\ h = 0.20 \text{ cm} \text{ to } 0.13 \text{ cm}.
\]

The tank was excited by the El Centro earthquake (1940) accelerogram, and the deformations shown in Fig. 8 are an example of the deformations measured at the top-rim. Fig. 8 shows that the circumferential and axial wave modes of the shell theory are strongly excited by seismic loads and that a circular cross-section is never observed at any of the recorded time steps. It has to be noted, that the analysed tank model had a deformability parameter \( K=0.173 \), calculated by a medium wall thickness, which is in the range of common existing tanks. In practice there are tanks with smaller \( K \)-values for which even higher deformations can be expected.

7. Damaged tanks

Due to lack of detailed information for tanks which have been destroyed in recent earthquakes, it is very hard to analyse the reasons for their failures. A good assembly of tank parameters is given in reference [12] for tanks that failed and for tanks that did not fail in four strong earthquakes. Although earthquake damage often

\[
\begin{align*}
\text{Fig. 7. } & \cos n\Theta \text{-type modes.} \\
\text{Fig. 8. } & \text{Top rim deformation history of a test tank.}
\end{align*}
\]
involves the roof and the piping connections, the only damage considered in the reference involves the shell.

Vague reasons are listed for the failure of the presented tanks such as bad soil conditions, seismic differential support settlement or a dynamic distortion of the shell cross-section.

If we analyse the presented tanks bearing in mind the properties of the deformability parameter $K$, interesting results can be achieved. If the parameter $K$ is calculated for the different tanks, and the sequence of tanks is reordered for rising $K$ values, the results of Table 5 are obtained.

Table 5 shows that independent from the earthquake magnitude and location, most of the damaged tanks are related to higher $K$ values. The two collapsed tanks represent almost the highest value of $K$, while the tank with the smallest $K$ value suffered no major shell damage, but an apparent bulge in the bottom ring. These results certify that the deformability parameter $K$ is a very useful indicator for the seismic design of liquid containing tanks, and that the commonly used ratios $L/r$ and $h/r$ cannot be considered separately from each other. Unfortunately the presented data is not precise enough in order to determine favourable and not favourable ranges for $K$. Some of the indicated wall thicknesses were only estimated in reference [12], and, in addition, the liquid height to tank height ratio is quite variable. Clarifying this is a task for further research projects. These results and the use of the deformability parameter $K$ could be a step towards “universal designing rules”.

To get more information about the reason for failures of the presented tanks and to check the results of the new model for the frequency analysis, the accelerograms of the four mentioned earthquakes can be analysed. For each of the accelerograms the corresponding Fast-Fourier-Transformation can be computed in order to get the frequency of ground motion for which the highest ground acceleration and therefore the highest energy

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Damage: B, shell buckled in bottom ring; C, shell collapsed; –, no damage; b, apparent bulge in bottom ring, no major shell damage.

a Alaska: 27.3.64
b Niigata: 16.6.64
c San Fernando: 9.2.71
d Kern County: 21.7.52.
occurred. For every tank, the fundamental frequency of the tank-liquid-system has been calculated three times. First with Eq. (2) which represents the frequencies proposed by Veletos, considered in API Standard 650, then with Eq. (1) which is the frequency proposed in Eurocode 8, and finally applying the new model presented in Section 5. These three frequencies are compared with the ground frequencies for which highest accelerations occurred at the corresponding earthquake. The results are presented in Table 6.

Analysing Table 6 the following results can be obtained:

1. The fundamental frequencies of the tank–liquid-system proposed by API Standard 650, by Eurocode 8 and by the new model are significantly different from each other.
2. The fundamental frequencies proposed by API Standard 650 and Eurocode 8 are much higher than the ground frequencies for which peak accelerations occurred, and not even lie in the range of the higher accelerations.
3. The fundamental frequencies proposed by the new model of Section 5 match very well with the ground frequencies for which peak accelerations occurred at the corresponding earthquakes.

Therefore, it is likely that resonance effects caused the failure of the analysed tanks. The results of Table 6 lead to another crucial detail for the seismic design of liquid storage tanks. Neither in Eurocode 8 nor in API Standard the case of resonance is taken into account. Certainly it is said that depending on the ground shaking frequency one of the three pressure components will be dominant and the others can be neglected. But a possible dynamic amplification in case of resonance is not taken into account! The fact that the frequency of the tank-liquid system is much lower than assumed for a long time, leads to the question whether the three pressure components are really independent from each other.

8. Non-linearity of stress distribution

As already discussed, the assumption that the tank reacts like a vertical cantilever beam without deformation of its cross-section is not convincing. Another debatable assumption adopted in Eurocode 8 and in API Standard is the assumption that the maximum wall pressures and the maximum shell stresses occur in the axis of seismic excitation as shown in Fig. 9.

Results of existing numerical methods have to be provided with high safety factors against many uncertainties. The real pipeline design, for example, is confronted with the use of safety factors up to ten. This is neither a good safety concept nor an economic approach.

In reference [13] stresses in cylindrical shells were analysed for fluid and granular filling and a Fourier series solution and its results are presented for various shell geometries.

In latter reference the deformability parameter $K$ and its relations to the stress distributions of horizontally placed simply supported thin circular cylindrical shells filled with water to any desired level are presented. In the design of these “pipes”, it was generally assumed that the maximum stress occurred at the bottom of the vertical diameter when the shell was half-filled. The paper shows that this is not necessarily true.

In fact it is shown that for $K \geq 2.0$ the stresses can be calculated by the simple beam equations regardless of the load level. For smaller $K$, in contrary, the stress distribution is highly non-linear as shown in Fig. 10 for different load levels. It is further shown that pipes with the same deformability parameter $K$ have identical stress profiles, independent of their dimensions. It is worth mentioning that the same $K$ can be obtained by completely different cylinders, one being long and slender, the other being short and bulky.

The reason of mentioning the analysis of the horizontally placed cylinders is the following. The presented problem can easily be transferred to the current problem of the vertical cylinders excited by horizontal ground motion.

In both cases, a perpendicular pressure is acting on the cylinder walls, even though at the upright cylinder the pressure distribution certainly is not as uniform as in the pipe case, depicted in Fig. 11.

For pipe design it has been shown, that the beam solution can only be used for $K > 2$ [13]. As common steel tanks seldom exceed a value of $K=0.4$, independent of their height to radius ratio, the assumption of the beam model and the above mentioned linear stress distribution should be avoided. In fact the actually resulting highly non-linear stresses should be calculated.

A convenient method to analyse these effects seems to be the same method as it is used for the horizontally placed cylinders, presented in [13, 14]. It is a new technique for calculating Fourier coefficients for one- or two-dimensional functions. The two-dimensional Fourier series are used to specify the load acting on the cylinder.

The presented results lead to the conclusion that for seismically excited upright cylinders the assumed stress distribution presented in Fig. 9 must be abandoned. In future research projects emphasis should be placed on the highly non-linear stress distributions being found in real tanks.

9. Conclusions

A novel model has been presented to calculate the fundamental frequencies of the tank–liquid system of a
### Table 6
Comparison between API Standard, Eurocode and New Model

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<th>Ratios</th>
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circular cylindrical tank. It has been shown that these frequencies match very well with the ground frequencies of earthquakes for which peak accelerations occurred. Resonance effects mainly caused the failure of tanks.

The question is to what extent this change of the fundamental frequency influences the design provisions of Eurocode 8? Changing the liquid–tank-frequency in the Eurocode 8 design process leads to a new maximum pseudo-acceleration $A(t)$ in the equation for the third pressure contribution called “Impulsive effect due to tank deformation”.

According to typical response spectra, a smaller fundamental frequency may lead to a higher but also to a smaller value for $A(t)$, and, therefore, a magnification or a reduction can be the consequence for this pressure component.

But, to abandon the cantilever beam model results in much greater consequences, because the current response spectra cannot be used any longer. These response spectra are only valid for a single degree of freedom oscillator and not for the highly complex system of a circular cylinder moving in its modal forms. To develop a corresponding response spectra with the aim to get the pseudo-acceleration for the assumed pressure distribution is a difficult task.

New questions result from the latter statement. Is it possible to develop such a response spectrum? The answer seems to be NO, because every point of the tank circumference seems to experience a different pseudo-acceleration when moving in its modal forms. Is it possible to use a model where all the liquid mass is simplified to a single lumped mass applying a specific pseudo-acceleration?

The answer must be NO, if we want to take into account the real tank behaviour and if we want to achieve more realistic results for the seismic design of tanks.

The basis for the presented model, to attach the impulsive mass component uniformly to the tank wall is certainly a simplification and might be improved in future work, but the presented results already show that it is the right direction.

Developing a new design method which takes into account all the mentioned items is still a challenge. But a big step is already made with a new mathematical method described in reference [14], the work presented here and the latest developments on base motion analysis of structures [15].

Future work will develop this improved model and this will reconsider other assumptions of current design standards. For example, if the assumption of the “rigid tank” pressure component is reasonable. We noticed, that common steel tanks cannot be regarded as rigid, as the $K$ deformability parameter lies within the range of 0.2 and 0.4, remember the strain patterns in Fig. 10. In the new model there is no need for the rigid tank pressure component, because the new model is based on the covariant and contravariant modal forms described in reference [14], and they are independent of the flexibility of the tank.

Finally, the only important conclusion of the presented results is to reconsider the current design provisions. Not only because their basic assumptions are no longer sustainable with the present knowledge of shell theory and shell design, but rather because the new results show that a high safety risk is induced by these provisions.
References